Polarization Precession Effects for Shear Elastic Waves in Rotated Solids

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Abstract
Developments of Solid-State Gyroscopy during last decades are impressive and were based on thin-walled shell resonators like HRG or CRG made from fused quartz or leuko-sapphire. However, a number of design choices for inertial-grade gyroscopes, which can be used for high-g applications and for mass- or middle-scale production, is still very limited. So, considerations of fundamental physical effects in solids that can be used for development of a miniature, completely solid-state, and lower-cost sensor look urgent.

There is a variety of different types of bulk acoustic (elastic) waves (BAW) in anisotropic solids. Shear waves with different variants of their polarization have to be studied especially carefully, because shear sounds in glasses and crystals are sensitive to a turn of the solid as a whole, and, so, they can be used for development of gyroscopic sensors.

For an isotropic medium (for a glass or a fine polycrystalline body), classic Lamé’s theorem (so-called, a general solution of Elasticity Theory or Green-Lamé’s representation) has been modified for enough general case: an elastic medium rotated about an arbitrary set of axes.

Travelling, standing, and mixed shear waves propagating in an infinite isotopic medium (or between a pair of parallel reflecting surfaces) have been considered too. An analogy with classic Foucault’s pendulum has been underlined for the effect of a turn of a polarizational plane (i.e., an integration effect for an input angular rate) due to a medium’s turn about the axis of the wave propagation. These cases demonstrate a whole-angle regime of gyroscopic operation.

Single-crystals are anisotropic media, and, therefore, to reflect influence of the crystal’s rotation, classic Christoffel-Green’s tensors have been modified.

Cases of acoustic axes corresponding to equal velocities for a pair of the pure-transverse (shear) waves have of an evident applied interest. For such a special direction in a crystal, different polarizations of waves are possible, and the gyroscopic effect of “polarizational precession” can be observed like for a glass.

Naturally, formation of a wave pattern in a massive elastic body is much more complex due to reflections from its boundaries. Some of these complexities can be eliminated. However, a non-homogeneity has a fundamental nature for any amorphous medium due to its thermodynamically-unstable micro-structure, having fluctuations of the rapidly-frozen liquid. For single-crystalline structures, blockness (walls of dislocations) plays a similar role.

Physical nature and kinematic particularities of several typical “drifts” in polarizational BAW gyros (P-BAW) have been considered briefly too. They include irregular precessions (“polarizational beats”) due to: non-homogeneity of mass density and elastic moduli, dissymmetry of intrinsic losses, and an angular mismatch between propagation and acoustic axes.

1. Introduction

Studies of fundamental effects of precession in polarizational states of shear elastic waves in solids have been forwarded from publically-closed reports to wide discussions by a scientific community as early as mid 80s at (2), at “Gulyaev’s conference” (2), and in article (3), etc. Similar effects have been studied also for anisotropic media, and have been discussed later at “Peshekhonov’s conference” (4) and in article (5). Selected aspects of gyroscopic sensing have been noted also in reports (6).
Actually, these effects\(^{(7)}\) have a lot of analogies with effects discovered by G.H.Bryan for oscillations of elastic shells\(^{(8)}\) and by E. Fermi for electromagnetic waves\(^{(9)}\).

Their math model forms a basis for studies of bulk acoustic waves (BAW) and for considerations of similar effects for surface acoustic waves (SAW)\(^{(10,11,12,13)}\). So, a number of patents, directly using these matters is rapidly increasing in different countries\(^{(14,15)}\),\(^{(16,17)}\), etc.

2. Basic Polarizational Effect

Pressure elastic ("acoustic") waves ("sounds", "ultrasounds", etc.) in fluids and solids are longitudinal waves. But shear elastic waves ("shear sounds") in glasses are transverse waves, and, therefore, they have different polarizations: linear (LP), circular (CW – clockwise or CCW – counter-clockwise), or combined (elliptic). There are many enough complex types of elastic waves in crystals due to their anisotropy of these media and a variety of syngonies (different classes of symmetry). On the other hand, polarizational effects in a crystal can be very similar to a glass, if an observer considers propagations along special directions (so-called, acoustic axes).

From a first glance, the basic effect looks simple: When a solid body is rotated as a whole about an acoustic axis, the transverse wave propagating in this direction is under an influence of Coriolis’ acceleration, and such a linear–polarized wave is precessing similar to well-known Foucault’s pendulum.

3. Modified Christoffel–Green’s Tensor

Acoustics of non-rotated crystals has been considered in numerous articles and several monographs (See, e.g., \(^{(18)}\), etc.). Studies of anisotropic elastic media intensively use formalism of tensors.

The classic Christoffel–Green’s tensors for an arbitrary anisotropic solid have been modified \(^{(4–6)}\) for a case of its rotation with an angular rate \(\ddot{\Omega}=(\Omega_i)\) about an arbitrary axis \(\hat{m}=(m_i)\):

\[
\Gamma'_s = \Gamma_s + \frac{\rho}{k^2} (\Omega_i - \Omega^2 \delta_i) = \Gamma_s + \Omega^2 (m_i m_i - \delta_i),
\]

where \(\dot{r}=(r_i)\) is the classic Christoffel–Green tensor of an unrotated medium having elastic constants \(\hat{c}=(c_{ijkl})\) and mass density \(\rho\). This tensor is defined for a wave propagating in a direction \(\hat{n}=(n_i)\).

![Figure 1 Precession of polarizational plane of a shear ultrasound propagating inside a rotated crystal or a glass sample.](image)

2. Analytical Solutions

If a solid has an axis of symmetry with equal phase velocities of quasi-transverse elastic waves (i.e., it has an acoustic axis)

\[
\nu'_s = \nu''_s ,
\]

this case is a subject of our special applied interest, because it can be used for development of miniature solid-state gyroscopic sensors.

4. Modified Lamé–Green’s Representation

Let us pay more attention to a case of an isotropic medium (i.e., an amorphous solid or polycrystalline one with fine and randomly-oriented grains). For such a case, the basic Navier’s equations of Elasticity Theory can be reduced to a system of wave equations (so-called, Lamé–Green’s representation theorem)\(^{(19,20)}\). This classic theorem has been