Fuzzy Regular Open and Closed Sets

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Abstract

In this paper we introduce and study the concepts of fuzzy regular open sets and fuzzy regular closed sets and by using them, we investigate the properties of the fuzzy almost continuous and fuzzy weakly continuous.

Zadeh[5] introduced the fundamental concept of a fuzzy set. This was further generalized by Goguen[4]. Algebraic properties of these sets were studied by De Luca and Termini[7]. Weaker forms of continuity in topology have been considered by many workers[1, 2, 5, 6, 8, 9, 10] using the concepts of semiopen sets, semiclosed sets, regular open sets and regular closed sets.

Let $I$ be the unit interval $[0, 1]$ of the real line. For any set $X$, $I^X$ denotes the collection of all mappings from $X$ into $I$. A member $A$ of $I^X$ is called a fuzzy set of $X$. The union $\cup A_a$ of a family $(A_a)$ of fuzzy sets of $X$ is defined to be the mapping $\sup A_a$ and the intersection $\bigcap A_a$ of a family $(A_a)$ of fuzzy sets of $X$ is defined to be the mapping $\inf A_a$. For any two members $A$ and $B$ of $I^X$, $A \supset B$ if and only if $A(x) \geq B(x)$ for each $x \in X$, and in this case $A$ is said to contain $B$, or $B$ is said to be contained in $A$. The complement $A'$ of a fuzzy set $A$ of $X$ is $1-A$ defined by $(1-A)(x)=1-A(x)$, for each $x \in X$.

Let $f: X \rightarrow Y$ be a mapping. If $A$ is a fuzzy set of $X$, we define $f(A)$ as $f(A)(y)=\sup_{x \in f^{-1}(y)} A(x)$ if $f^{-1}(y) \neq \emptyset$ or $f(A)(y)=0$ if otherwise, for each
\( y \in Y \) and if \( B \) is a fuzzy set of \( Y \), we define \( f^{-1}(B) \) as \( f^{-1}(B)(x) = (A \cdot f)(x) \) for each \( x \in X \).

**Definition 1.** A subfamily \( \mathcal{J} \) of \( I^X \) is called a fuzzy topology on \( X \) if (1) 0 and 1 belong to \( \mathcal{J} \), (2) any union of members of \( \mathcal{J} \) is in \( \mathcal{J} \), and (3) a finite intersection of members of \( \mathcal{J} \) is in \( \mathcal{J} \).

Members of \( \mathcal{J} \) are called fuzzy open sets of \( X \) and their complements fuzzy closed sets. A system \((X, \mathcal{J})\) consisting of a set \( X \) with fuzzy topology \( \mathcal{J} \) on \( X \) is called a fuzzy space \( X \).

**Definition 2.** For a fuzzy set \( A \) of \( X \), the closure of \( \text{Cl} A \) and the interior \( \text{Int} A \) of \( A \) are defined respectively, as \( \text{Cl} A = \inf \{ B \mid B \supseteq A, B \in \mathcal{J} \} \) and \( \text{Int} A = \sup \{ B \mid B \subseteq A, B \in \mathcal{J} \} \).

The next theorem is followed from the Theorem 2.13 in [11].

**Theorem 3.** For a fuzzy set \( A \) of a fuzzy space \( X \), (1) \( 1 - \text{Int} A = \text{Cl}(1 - A) \), and (2) \( 1 - \text{Cl} A = \text{Int}(1 - A) \).

**Definition 4.** A fuzzy set \( A \) of a fuzzy space \( X \) is called (1) a fuzzy regular open set of \( X \) if \( \text{Int} \text{Cl} A = A \), and (2) a fuzzy regular closed set of \( X \) if \( \text{Cl} \text{Int} A = A \).

According to the Theorem 3, we get the following result.

**Theorem 5.** A fuzzy set \( A \) of a fuzzy space \( X \) is fuzzy regular open if and only if \( A' \) is fuzzy regular closed.

**Theorem 6.** (1) The intersection of two fuzzy regular open sets is a fuzzy regular open set.

(2) The union of two fuzzy regular closed sets is a fuzzy regular closed set.

**Proof.** (1) Let \( A \) and \( B \) be any two fuzzy regular open sets of a fuzzy space