A Dynamic Model with Backlogging for Inbound Transportation at a Third-Party Warehouse

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Abstract

We consider a lot-sizing model with dynamic demands for inbound shipment consolidation and replenishment schedule at a third-party warehouse. To consider service terms on waiting for an agreed upon amount of time or on some portion of money-back guarantee in 3PL industry we allow backlogging. For this model, an $O(T^2)$ algorithm is designed.

1. Introduction

One of the most important methods in realizing the economies of scale in third-party logistics (3PL) is freight consolidation. The freight consolidation practice increases the utility of fleet assets like trucks, railroads, or air crafts by minimizing the case of less-than-truck load (LTL) but maximizing the chance of full-truck load (FTL) delivery. The decisions in making replenishment schedule with shipment consolidation are on balancing the trade-offs between the opportunity to increase the chance of FTL delivery and the objective to minimize inventory: We carry items forward to the next periods to accumulate and then pack them in full capacity, which may decrease LTL deliveries but at the same time it increase the inventory holding cost.

In this paper we consider a typical 3PL provider engaged in partnership with production firms performing shipment consolidation on behalf of them. Manufacturers in neighborhood puts their products to a common place called center and then the 3PL provider collects, consolidates them into cargos and transports cargos to the main warehouse called hub. As customer satisfaction is a key success factor, the 3PL provider needs to cope with volatile small sized orders requiring door-to-door services. To incorporate such orders with the advantage of freight consolidation, it is often the case that the 3PL provider set service terms on waiting for an agreed upon amount of time with some discount in transportation rates, or some portion of money-back guarantee if 3PL fails to meet the delivery commitments.

The study on freight consolidation started in logistics literature. It has been reported that we can lower transportation costs using freight consolidation practice (Newbourne and Barrett 1972, Masters 1980, Jackson, 1985). Policies on the location of the centers and the storage amount it should hold before loading are suggested (Gupta and Bagchi 1987, Higginson and Bookbinder 1994). To deal with the trade-offs between consolidation and inventory, Cetinkaya and Lee (2000, 2002) provided analytical models with stochastic demands. In the literature, however, most research in inbound freight consolidation has focused on the usage of carrying inventory. Until now, not much study has been done for backlogging inventory except for Lee et al. (2003) allowing it for outbound customers.

In this paper, we consider a dynamic lot-sizing model for inbound transportation at a third-party warehouse allowing backlogging. With the strategic alliance program between players in supply chain, planners can now make schedules based on known dynamic demands (Lee 2001, Hwang 2006a, b). This motivates the deterministic model in his paper. In contrast with service industry like 3PL, a large amount of research in dynamic lot-sizing model has been done in production industry. With the advent of seminal paper of Wagner and Whitin (1958), dynamic lot-sizing models were studied to explain various situations, in particular, production with unlimited and limited cases: For production with unlimited capacity see Zangwill (1966 and 1969), Federgruen and Tzur (1991), Wagelmans et al. (1992), Aggarwal and Park (1993); For production with limited capacity see Florian and Klein (1971), Love (1973), Swoveland (1975), Florian et al. (1980), Bitran et al. (1984), Chung and Lin (1988), Chen et al. (1994), Hoesel and Wagelmans (1996).
with Lippman (1969) in the name of multiple setup lot-sizing model. This was generalized to take into account setup cost in replenishment (Lee 1989). Jaruphongsa et al. (2005) provided optimal replenishment procedures for various shipment modes under the freight consolidation practice. Lee (2004) extended his result (1989) to deal with replenishment model for decision making on trade-offs between lot-sizing and just-in-time delivery. In Lee (2004), an $O(T^3)$ algorithm was proposed. Recently, Hwang (2006c) explored the phenomena of economies of scale in 3PL industry and suggested a model to take full advantage of it. He considered a lot-sizing model with inbound transportation with concave replenishment and inventory costs. He provided optimal $O(T^3)$, $O(T \log T)$ and $O(T^2)$ algorithms for concave, fixed-plus-linear and nonspeculative cost structures, respectively. This paper is a direct extension of Hwang (2006c) in which backlogging is allowed. For this problem we present an $O(T^2)$ algorithm for nonspeculative cost structure.

In the next section, our model will be formally defined and optimality properties be presented. The $O(T^2)$ optimal procedure is developed in Section 3. In the final section, we conclude this paper.

2. Model Formulation and Optimality Properties

Let $T$ denote the length of planning horizon. For each period $t \in \{1, 2, \ldots, T\}$ we define:

- $d_t$: demand in $t$.
- $C$: unit cargo capacity.
- $A$: unit cargo delivery cost.
- $x_t$: replenishment level in $t$.
- $I_t^r$: inventory level in $t$.
- $I_t^b$: backlogging level in $t$.
- $p(x_t)$: replenishment cost in $t$ for the amount $x_t$.
- $h_t(I_t^r)$: inventory holding cost in $t$ for the amount $I_t^r$.
- $b_t(I_t^b)$: inventory backlogging cost in $t$ for the amount $I_t^b$.

Under the nonspeculative cost structure with stationary setup cost, the functions $p(x_t)$, $h_t(I_t^r)$ are given as:

$$p(x) = K + p_0 x, \quad h_t(x) = h_0 x, \quad \text{and} \quad b_t(x) = b_0 x,$$

where $K$ is setup cost, $p_0$, $h_0$ and $b_0$ are unit replenishment, holding and backlogging costs in period $t$, respectively. Furthermore, we assume that $p_0 + h_0 \geq p_{t+1}$ and $p_0 \leq p_{t-1}$ for nonspeculative cost structure.

In general, the replenishment and inventory cost functions $p(I_t)$, $h_t(\cdot)$ and $b_t(\cdot)$ are assumed to be concave. We let $\lfloor x \rfloor$ and $\lceil x \rceil$ denote the smallest integer no less than $x$ and the largest integer no greater than $x$, respectively. To make it easy in treating the quantities related with cargo capacity we define the following simple functions:

- $\pi(x)$: minimum number of cargos to carry the amount of $x$; $\pi(x) = \lfloor x/C \rfloor$.
- $n(x)$: minimum number of FTL cargos to carry the amount of $x$; $n(x) = \lceil x/C \rceil$.
- $m(x)$: number of items carried in FTL cargos for the amount of $x$; $m(x) = \lceil x/C \rceil C$.
- $\Delta(x)$: number of items carried in LTL cargos for the amount of $x$; $\Delta(x) = x - m(x)$.

The mathematical formulation of the problem is given by:

$$\begin{align*}
\text{Min} & \quad \sum_{t=1}^{T} (\pi(x_t) \cdot A + p_t(x_t) + h_t(I_t^r) + b_t(I_t^b)) \\
\text{Subject to} & \quad (I_{t+1}^r - I_{t-1}^r) + x_t = d_t + (I_t^r - I_t^b), \quad t = 1, K, T \\
& \quad I_0 = I_T = 0, \\
& \quad x_t \geq 0, I_t^r \geq 0, \quad t = 1, K, T \\
& \quad I_t^b \geq 0, \quad t = 1, K, T
\end{align*}$$

The following property can be easily derived based on the fractional production property in the classical capacitated lot-sizing problem (Florian and Klein 1971). A period $t$ is called a replenishment period if we have a replenishment in period $t$, i.e., $x_t > 0$. For a replenishment period, if $x_t = m(x_t)$, then $t$ is said to be an FTL (full-truck load) period, otherwise an LTL (less-than-truck load) period.

**Property 1.** There exists an optimal solution such that if $\lambda - 1$ and $\gamma$ are two consecutive regeneration periods, i.e., $I_{\lambda - 1} = I_\gamma = 0$ and $I_t \neq 0$ for $t = \lambda, \lambda + 1, \ldots, \gamma - 1$, then $x_t$ is either zero or at FTL in every period $t$, except in at most one.

Applying the result developed in Lee (1989), we also obtain the following property.

**Property 2.** Under a nonspeculative cost structure with stationary setup cost, there exists an optimal solution such that $x_t > 0$ if and only if $I_{\lambda - 1} < \min \{C, d\}$ and $I_{\gamma + 1} < \min \{C, d\}$ for every $1 \leq t \leq T$.

Then, by Property 1 and Property 2 with the fact every replenishment is FTL except at most one, we can show the next property.

**Property 3.** Suppose that $I_{\lambda - 1} = I_\gamma = 0$ and any optimal solution keeps inventory throughout the periods, i.e., $I_t \neq 0$, $t = \lambda, \lambda + 1, \ldots, \gamma - 1$. We further suppose that there is an LTL replenishment during periods $s, s+1, \ldots, t$, $\lambda \leq s \leq t \leq \gamma$.