A Branch-and-price Approach to the ATM Switching Node Location Problem

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Abstract. We consider the ATM switching node location problem (ANLP). In this problem, there are two kinds of facilities, hub facilities and remote facilities, with different capacities and installation costs. We are given a set of customers with each demand requirements, a set of candidate installation sites of facilities, and connection costs between facilities. We need to determine the locations to place facilities, the number of facilities for each selected location, the set of customers who are connected to each installed hub via installed remote facilities with minimum cost, while satisfying demand requirements of each customer. We formulate this problem as a general integer programming problem and solve it to optimality. In this paper, we present a preprocessing procedure to tighten the formulation and develop a branch-and-price algorithm. In the algorithm, we consider the integer knapsack problem as the column generation problem. Computational experiments show that the algorithm gives optimal solutions in a reasonable time.

Keywords: Facility Location, Integer Programming, Column Generation, Branch-and-Price

1. INTRODUCTION

In the past few years, many experiments to implement broadband integrated services digital network (B-ISDN) have been conducted. To support these B-ISDN service requirements, ATM (Asynchronous Transfer Mode) has been proposed as the target technology. ATM is a packet-oriented transfer mode in which information is organized into a fixed-size entity known as a cell. ATM technology combines the flexibility of traditional packet-switching technology with the determinism of TDM (time division multiplexing) (Wu 1990).

To carry these flexible B-ISDN services, we should install ATM switching nodes. In this paper, we consider the ATM-MSS (ATM-MAN Switching System) Node Location Problem (ANLP) for the PVC (Permanent Virtual Connection) based leased line network. In this network, services are provided at a constant bit rate (CBR) or variable bit rate (VBR). In this paper, the QoS and/or statistical multiplexing are considered through the equivalent cell rate (ECR).

First, we consider the switching systems called the ATM-MSS switching nodes. We are given two kinds of
facilities: Hub Switching Node (HSN) and Remote Switching Node (RSN). Each HSN accommodates several RSNs in the star topology. Each RSN accommodates user demands with various interfaces such as DS1E (2.048 Mbps), DS3 (44.736 Mbps) and STM-1 (155.520 Mbps), with the capacity of 284 DS1E. According to these functions, we may call the HSN the hub facility and the RSN the remote facility. For each candidate site of facilities, we may install more than one facility.

Then, the ANLP is defined as follows. We are given hub candidate sites \( H \), remote candidate sites \( R \) and users \( U \). Each user \( u \) should be connected to the remote facilities installed at one remote candidate site to satisfy demand requirement \( r_u \). The remote facilities connected to a user should be connected to the hub facilities installed at one hub candidate site. Each hub facility has a finite capacity \( b_h \) and a fixed cost \( F_{bh} \) and each remote facility has a finite capacity \( b_r \) and a fixed cost \( F_{br} \). Connection cost \( d_{ur} \) arises when the unit demand \( u \) is satisfied by the flow from the remote facility installed at a remote candidate site \( r \). The connection cost \( d_{hr} \) arises when a remote facility installed at a remote candidate site \( r \) is connected to the hub facility installed at a hub candidate site \( h \).

Then, we determine the number of hub facilities and the number of remote facilities for each candidate site and the allocation of users to hub candidate sites via remote candidate sites with minimum total cost, the sum of facility installation cost and connection cost.

Facility location problems have received a great deal of attention for recent several decades. For the general capacitated plant location problem, Sa (1969), Davis and Ray (1969), Ellenwein and Gray (1971), Akinc and Khumawala (1977) have studied. But in most formulation of facility location problems, a single stage distribution system has been considered.

For the two stages distribution system where commodities are delivered from plants to customers via warehouses, Geoffrion and Graves (1974) considered a multicommodity two stage problem with the restriction that each customer should be served by only one warehouse. Tcha and Choi (1980) also studied of the single commodity two stages problem without aforementioned restriction. But since customers and plants are given, they determined warehouse locations only.

Kaufman et al. (1977) have studied of the problem of location simultaneously both plants and warehouses with no capacity restrictions. For the problem with single assignment restriction, Neebe and Rao (1983), Dee and Lieman (1972), Barcelo and Casanovas (1984), and Tang et al. (1978) are widely known.

Recently, the cutting plane method using polyhedral structure has become a standard technique. Survey on this method can be found in Nemhouser and Wolsey (1988). Crowder et al. (1982) and Johnson et al. (1985) reported the success of solving large scale 0-1 programming problems arising from planning models using this method. Especially Aardal et al. (1995) and Leung et al. (1989) have derived some valid inequalities for the capacitated facility location problem. Delayed column generation approach incorporated with the branch-and-bound procedure has also become a new technique for solving the combinatorial optimization problem. For example, Savelsbergh (1997) has reported the success of solving the generalized assignment problem.

In this paper, we use the delayed column generation and branch-and-price approach. We formulate this problem using tree variable. To solve the linear programming relaxation of the formulation, which has exponentially many variables, we solve the pricing subproblem, which is NP-hard. But we can solve the subproblem using the pseudo polynomial-time algorithm. Moreover, to tighten the bound of LP relaxation, a preprocessing procedure is devised by deriving some valid inequalities.

The remainder of the paper is organized as follows. In section 2, we state the notations for the problem description and formulate the problem using the concept of pattern generation. In section 3, the column generation subproblem is considered. In section 4, we present a preprocessing procedure to tighten the bound of LP relaxation of problem and present the augmented linear programming. In section 5, we present the branch-and-price algorithm and implementation details. In section 6, we show the computational results of our algorithm. Finally, we give concluding remarks.

## 2. FORMULATION OF THE PROBLEM

In this section, we present the formulation of ANLP. Since each user should be connected to a hub candidate site via a remote candidate site, we can allocate some users to be connected to a hub candidate site via a remote candidate site. For the fixed hub candidate site \( h \) and the fixed remote candidate site \( r \), there are several kinds of allocation patterns. We call this allocation pattern as a tree. Then ANLP finds the set of trees and an assignment of facilities for each selected hub candidate site and the selected remote candidate site to minimize the cost while satisfying the demand requirements of the users.

First, we give some notation to be used in the formulation of the problem.

- \( NR(h) \) : set of remote candidate sites that can be connected to hub candidate site \( h \).
- \( NU(rh) \) : set of users that can be connected to hub candidate site \( h \) via remote candidate site \( r \).
- \( T(rh) \) : set of feasible trees rooted at hub candidate site \( h \).