A Method of Evaluating Profitability and Risk of Multiple Investments Applying Internal Rate of Return

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Abstract. In today’s uncertain economic environment, economic risk is inherent in making large investments on manufacturing facilities. It is, therefore, practically meaningful to divide investment over multiple periods, reducing the risk of investment. Then, the cash-flow over the entire planning horizon would comprise positive inflow and negative outflow. In this case, in general, evaluation by internal rate of return (IRR) is not feasible, because multiple IRRs are involved. This paper deals with a problem of evaluating profitability, as well as risk, of investment alternatives made in multiple times of investment over the entire horizon. Typically, an additional investment is required after the initial one, for expanding manufacturing capacity or other reasons. The paper pays attention to a unit cash-flow over two periods, decomposing the total cash-flow into a series of unit cash-flow patterns. It is easy to evaluate profitability of a unit cash-flow by using IRR. The total cash-flow can be decomposed into the series of two types of unit cash-flows: an investment type one (negative-positive) and the borrowing type one (positive-negative). This paper, therefore, proposes a method in which only the borrowing type unit cash-flow is eliminated in the series by converting total cash-flow using capital interest rate. Then, a unique IRR can be obtained and the profitability is evaluated. Thus, the paper extends the method of IRR so that it may help decision making in complicated cash-flow pattern observed in practice.

Keywords: Profitability, Risk, Unit Cash-flow, Internal Rate of Return, Multiple Investment

1. INTRODUCTION

Under recent uncertain economic environment, it is sometimes reasonable in practice to divide investment into multiple times, in such a case as establishing an overseas manufacturing site. It means that initial investment is followed by another one several years later for the purpose of installing additional machinery or starting full capacity operation. In such a case, total cash-flow involves multiple times of investment, and therefore, so-called the internal rate of return (IRR) method is not applicable to judging profitability over the total life, since multiple IRRs may be obtained.

Many studies have been made on the issue of multiple IRRs. Such studies have focused on mathematical analysis of net present value (as well as net future value) function and IRR as roots of an equation of high degree. Teichroew et al. (1965) classify investment projects into pure investment and mixed investment. The phenomenon of multiple IRRs occurs only in the situation of a mixed investment. They propose the project investment rate (PIR) which is dependent on the capital interest rate. The mixed investment can be judged by comparison of PIR and the capital interest rate. As a trial and error approach, a set of computer codes for calculating PIR are proposed (Ward, 1994). Mizumachi et al. (2002) developed another algorithm for computing PIR as IRR of certain pure investment obtained by conversion of the original mixed investment based on the capital interest rate. Nevertheless, the proposed algorithm is unsatisfactory in the sense that the calculation of IRR in the mixed investment which might involve multiple IRRs still remains undeleted in the algorithm.

In this paper, we focus on one year investment project which consists of an investment (negative cash-flow) and a positive return. The IRR of one year investment project is calculated as the rate of increase of capital in the project. The project is clearly evaluated by the comparison of its IRR and the capital interest rate. We attempt to apply the IRR criterion of the one year investment project to the evaluation of the investment project over the entire horizon. We decompose total cash-flow of the investment project into a series of unit cash-flows over two periods. The unit cash-flow is classified into two types; the “investment type” in which negative cash-flow (investment) occurs at the end of one period...
and positive cash-flow (return) comes at the end of the next period, and the “borrowing type” in which positive cash-flow occurs at the end of one period and negative cash-flow follows at the end of the next one. We also analyze the cash-flow as a repetition of fundamental cash-flow pattern, namely, one time investment followed by positive returns. It is known that the fundamental cash-flow pattern has a unique IRR.

The practical concerns are: whether the IRR method is applicable to given cash-flow; and how it can be evaluated if the IRR method is not applicable. Major purposes of this paper are as follows:

1. To clarify the condition where IRR method is applicable to specific cash-flow pattern which involves two times of investment.
2. In case where the IRR method is not applicable, to present a new evaluation method applying IRR. In the method, calculation of multiple IRRs should be avoided.

Application of the procedure to more complicated cash-flow is an area for future study.

The rest of the paper is organized as follows. Chapter 2 presents the model, with assumptions and notations. Chapter 3 provides the fundamental analysis of the profitability of investments by decomposing the total cash-flow into the series of unit cash-flows. Chapter 4 presents the expanded IRR method. The numerical examples are discussed in Chapter 5. Finally the conclusion is presented in Chapter 6.

2. MODEL PRESENTATION

2.1 Assumptions and Notations

This paper investigates the problem using following assumptions and notations:

1. \( A = [a_0, a_1, \ldots, a_n] \) denotes an investment project which generates net cash flows of \( a_t \) at the end of period \( t \), where \( t = 0, 1, \ldots, n \).
2. The project balances of \( A \) for interest rate \( i \) are defined as follows:

\[
\begin{align*}
S^A_0 &= a_0 \\
S^A_t &= S^A_{t-1}(1+i) + a_t, \quad 1 \leq t \leq n 
\end{align*}
\]  

(1)

The project balance at the end of period \( n \) is equivalent to the Net Future Value (NFV) function.

\[
S^A_n = a_0(1+i)^n + a_1(1+i)^{n-1} + \cdots + a_{n-1}(1+i) + a_n
\]  

(2)

3. The IRR of \( A \) is denoted by \( r^A \), which satisfies

\[
S^A_i(r^A) = 0
\]  

(3)

4. The capital interest rate is given and represented by \( i_0 \).

5. We define a unit cash-flow over two periods. The unit cash-flow consists of \( c_t \) at the end of period \( t-1 \) and \(-c_t(1+r)\) at the end of period \( t \). We denote the unit cash-flow in period \( t \) whose IRR is \( r \) by,

\[
c^A_{i_0} = [0, \ldots, 0, c_t, (1+r), 0, \ldots, 0] = [c_t, -c_t(1+r)]
\]  

(4)

The subscript \( t \) outside of \([ \ ]\) represents the final period of the cash-flows listed in \([ \ ]\). As shown in Fig.1, the case where \( c_t < 0 \) is called that of the investment unit cash-flow, and the case where \( c_t > 0 \), that of the borrowing unit cash-flow.

Investment unit cash-flow

Borrowing unit cash-flow

**Figure 1.** Unit cash-flow.

6. We call the cash-flow which consists of one time investment and positive returns, the fundamental cash-flow pattern.

\[
\begin{align*}
&\quad a_0 < 0 \\
&\quad a_t > 0, \quad 1 \leq t \leq n 
\end{align*}
\]  

(5)

2.2 Problem to be Solved

This paper investigates the cash-flow involving investment in two times over the entire horizon. We denote the cash-flow pattern by \( D \). We define the judgment criteria for evaluating the profitability of \( D \) based on the comparison of IRR and the capital interest rate.

\[
D = [d_0, d_1, \ldots, d_k, \ldots, d_n], \quad \begin{cases} 
&d_0 < 0 \\
&d_k < 0, \quad (1 < k < n) \\
&d_i > 0, \quad (t \neq 0, k) 
\end{cases}
\]  

(6)

**Figure 2.** Cash-flow pattern of multiple investments.

Where \( D \) and the capital interest rate \( i_0 \) are given, the problems are:

1. To clarify the condition that \( D \) is classified into the pure investment to which IRR method is applicable.
2. When \( D \) is classified into the mixed investment, to present