One-sided Readings of Numbers in Modal Sentences*

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I. Introduction

Traditionally, an interpretation domain consists of individuals (and possibly materials), and nouns and predicates are understood to denote sets of individuals (or materials) in this domain. Depending on theories, additional entities of events or situations are required to construe verbal predicates. No matter how diverse entities an interpretation domain includes, all these entities are defined by inclusion. A larger set includes smaller sets, which reflects relative specificity of meanings between expressions.

Unlike nouns and predicates, the interpretations of number words cannot be done by the set inclusion of entities. The meaning of five cannot be defined by the inclusion of lower numbers such four or three. Moreover, numbers are in an ordering relation. Thus, an issue to deal with number interpretations is what entities are needed and how they are structured. Additionally, we need to probe into what numeral readings number words have. Basically, number words without any modification are interpreted in two ways: ‘exactly readings’ and ‘at-least readings.’ For example, the number three is construed either as ‘exactly three’ or ‘at least three,’ and the sentence in (1a) means either (1b) or (1c).

(1) a. The students registered three classes.
    b. The students registered exactly three classes.
    c. The students registered at least three classes.

In the exactly reading, both the upper and lower bounds are specified by the number, so it is also called a ‘two-sided’ reading. In the at-least reading, only the lower bound is restricted by the number, which makes it called a ‘one-sided’ reading.

* I wish to thank two anonymous reviewers for their helpful comments and suggestions. All remaining errors, however, are mine.
No matter which reading a number word takes, its interpretation is varied in a sentence with a modal operator. Allow and require are deontic modal operators that quantify over goal or ideal worlds. Allow is an existential quantifier on worlds while require has universal quantificational force on worlds. When a number occurs in a sentence with a modal operator, its exactly reading is not licensed.

(2) a. The students are allowed to register three classes.
   ≡ The students are allowed to register at most/maximally three classes.

b. The students are required to register three classes.
   ≡ The students are required to register at least/minimally three classes.

In both (2a) and (2b), three is not interpreted as ‘exactly three.’ Three has contrasted readings in the sentences other than this common property. Three in (2a) is construed as upper-bounded and three in (2b) lower-bounded as paraphrased. Given the fact that (2a) and (2b) are minimally differentiated from (1a) with the insertion of the modal operators, the distinct readings of three in (2) are attributed to the interaction of the modal operators and the number word. Note that meaning interactions of number words are not confined to modal operators. When a negative operator occurs with a sentence without a number, it does not affect the acceptability of the sentence. However, occurring in a sentence with a modal operator and a number, the acceptability of the sentence may be different depending on the scope of negation.

To trace the interaction between number words and modal and negative operators, I will consider the interpretations of number words in semantic literature. Much theoretical and experimental evidence shows that numbers are uniformly interpreted as two-sided. Based on the exactly readings of numbers, I will critically review a previous analysis on number readings in sentences with operators, and propose an alternative analysis in the framework of Vector Space Semantics.

II. Interpretations of Numbers in Modal Sentences

2.1 Interpretations of Number Words

The semantics of cardinal number words has been considered as no different from other scalar quantifiers such as some and most. Just as some is lower-bounded and the cases for all are excluded by scalar implicature,