Mathematical Explanation for the Wide and Deviated Range of Optimal Hematocrit

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Hematocrit is an important determinant of oxygen delivery. Of particular interest, its target level is very wide for different kinds of shock: from 30% for hemorrhagic or septic shock to 56% for secondary polycythemia. This range is not only wide but also deviated to the higher level from the optimal value of 40%. In this letter, the authors determine the mathematical basis of the wide and deviated range of hematocrit starting from the Hagen-Poiseuille equation.

Key Words: Hematocrit, Hemoglobin, Oxygen, Polycythemia, Shock

Shock is defined as “failure to deliver and/or utilize adequate amounts of oxygen”¹. Oxygen delivery (DO₂) is calculated as “13.4 × Hb × SaO₂ × ΔP × (πr⁴/8 μL)”. (Hb, hemoglobin concentration; SaO₂, O₂ saturation of hemoglobin; ΔP, pressure gradient along the vessel; r, inner radius of the vessel; μ, blood viscosity; L, length of the vessel) The current treatment of shock is focused on maximizing DO₂ by boosting its components. Regarding the component of hematocrit (Hct), which is usually 3 times of Hb, the early goal-directed therapy (EGDT) for severe sepsis and septic shock and the guidelines for hemorrhagic shock recommend transfusion if Hct is less than 30%, while they recommend phlebotomy for patients suffering polycythemia secondary to hypoxic pulmonary disease when Hct is more than 56%²-⁴.

In 1967, Crowell JW reported that blood viscosity (μ) can be expressed in terms of Hct like following: μ=k × e₀.₀₂₅Ho (k: a constant)⁵. By substituting this equation, DO₂ above can be expressed as a function of Hct by ignoring other parameters which are independent of Hct: DO₂=K × Hct/e₀.₀₂₅Ho (K: another constant). With this equation, he showed that DO₂ reaches its maximum when Hct is 40⁵.

Here come the following questions: (1) Why is the practical range of optimal Hct so wide from 30% for septic or hemorrhagic shock to 56% for secondary hypoxia with hyperviscosity? (2) Why is the range more deviated toward the higher level of the Hct from the reference value of 40%?

To get the answers, we drew the DO₂-Hct relationship plot (Fig. 1). The graph is asymmetrically ∩-shaped. As proven by Crowell already, DO₂ reaches the maximum (DO₂ max) when Hct is 40. We supposed that the optimal DO₂ exceed the ‘95%’ of DO₂ max. This optimal DO₂ is achieved when Hct falls in the range of 28.5~ 54.2% (Fig. 1). To revisit the 2 questions above, (1) the theoretical range of optimal Hct (28.5~ 54.2%) almost coincides with the practical one (30~56%) assuring its wide range, and (2) the left-skewed shape of the DO₂-Hct plot explains why the range of optimal Hct is deviated toward its higher level from the reference value of 40%.

In conclusion, the wide and deviated range of optimal hematocrit for shock patients could be explained mathematically by analyzing the oxygen delivery (DO₂) as the function of the hematocrit (Hct).

REFERENCES

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![Graph showing relative oxygen delivery (DO₂) as a function of hematocrit (Hct).](image)

**Fig. 1.** Relative oxygen delivery (DO₂) as a function of hematocrit (Hct).

DO₂ can be expressed as $K \times \text{Hct}^{0.025}\text{Hct}$. When drawn, it shows an asymmetric \( \cap \) shaped curve, reaching its maximum when Hct is 40. DO₂ exceeds the ‘95%’ of the maximum value (DO₂\(_\text{max}\)) when Hct falls in the range of 28.5–54.2%.