A note on fuzzy RS-compact spaces

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퍼지 RS-컴팩트에 관한 연구

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要 約

본 논문은 퍼지 위상공간에 대한 퍼지 RS-컴팩트성을 정의하고, 몇 가지 성질을 조사 연구 하였다.

1. INTRODUCTION AND PRELIMINARIES

The concepts of fuzzy almost compactness and fuzzy near compactness for fuzzy topological spaces were first introduced by Di Concillo and Gerla [5] and Es [6], respectively. Subsequently several authors [8, 11, 13, 14] further investigated some properties of these concepts in fuzzy topological spaces, using regular open sets and fuzzy closed sets. Recently, Mukherjee and Ghosh [13] introduced the concept of fuzzy S-closedness, which is independent of that of fuzzy compactness, and showed that every fuzzy extremally disconnected and fuzzy almost compact space is fuzzy S-closed.

In this paper we introduce and study fuzzy RS-compactness for fuzzy topological spaces.

Throughout this paper $X$ means fuzzy topological space (fts, for short) in Chang's [3] sense. For a fuzzy set $A$ of a fts $X$, the notations $\text{Cl}_A$, $\text{Int}_A$ and $1-A$ will respectively stand for the fuzzy closure, fuzzy interior and complement of $A$. By $0_X$ and $1_X$ we will mean the constant fuzzy sets taking on respectively the values 0 and 1 on $X$.

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Definition 1. A fuzzy set $A$ of a fts $X$ is said to be

(a) fuzzy regular open \([1]\) if $A = \text{Int} \overline{A}$.

(b) fuzzy semiopen \([1]\) if there exists a fuzzy open set $U$ such that $U \subseteq A \subseteq \text{Cl}U$.

(c) fuzzy regular semiopen \([13]\) if there exists a fuzzy regular open set $U$ such that $U \subseteq A \subseteq \text{Cl}U$.

It is obvious that every fuzzy regular open set is a fuzzy regular semiopen set and every fuzzy regular semiopen set is a fuzzy semiopen set. The Example 2.6 in \([13]\) and following example show that none of the converses need true in general. Also the following example show that fuzzy regular semiopen set and fuzzy open set are independent.

Example 2. Let $A$ and $B$ be fuzzy sets of $X = \{a, b, c\}$ defined as following:

- $A(a) = 0.4$, $A(b) = 0.3$, $A(c) = 0.2$.
- $B(a) = 0.5$, $B(b) = 0.6$, $B(c) = 0.4$.

(a) If we consider the fuzzy topology $\tau_1 = \{1_X, 0_X, A\}$ on $X$, then $A$ is fuzzy regular open set such that $A \subseteq B \subseteq \text{Cl}A$. Thus $B$ is fuzzy regular semiopen set in $X$. But $B$ is neither fuzzy open nor fuzzy regular open in $X$.

(b) If we consider the fuzzy topology $\tau_2 = \{1_X, 0_X, B\}$ on $X$, then $B$ is fuzzy open set. But $B$ is not fuzzy regular semiopen set, since there is no fuzzy open set $C$ such that $C \subseteq B \subseteq \text{Cl}C$.

Lemma 3. For a fuzzy regular semiopen set $A$ of a fts $X$, the following are true:

(a) $1-A$ is fuzzy regular semiopen.
(b) $\text{Int}A = \text{Int} \overline{A}$.
(c) $\overline{A} = \text{Cl} \text{Int}A$.

Proof. (a): Let $A$ be a fuzzy regular semiopen. Then there exists fuzzy regular open