Jeffrey's Noninformative Prior
in Bayesian Conjoint Analysis

Man-Suk Oh¹ and Yura Kim²

ABSTRACT

Conjoint analysis is a widely-used statistical technique for measuring relative importance that individuals place on the product's attributes. Despite its practical importance, the complexity of conjoint model makes it difficult to analyze. In this paper, we consider a Bayesian approach using Jeffrey's noninformative prior. We derive Jeffrey's prior and give a sufficient condition under which the posterior derived from the Jeffrey's prior is proper.

Key Words: Conjoint model, Proper posterior, Marketing research, Improper prior.

1. Introduction

Over the past three decades, conjoint analysis has evolved as a primary set of techniques employed by both academics and practitioners of marketing research for measuring consumer tradeoffs among multi-attributed products and services. It is a useful statistical technique for measuring the relative importance that individuals place on attributes of a product or service. In a typical conjoint study, various levels of several key attributes are selected, and product or service profiles are constructed using a level of each attribute. A survey is conducted and respondents are asked to rank the product or service profiles according to their preference. The observed rankings are then used to determine the relative impact of each feature on the individual's overall preference.

An objective of conjoint analysis is to assign a weight, called a partworth, to each level of each feature so that the ranking of the profiles based on the summation of the corresponding partworths reproduces the participant's original overall preference ranking.

¹This work was supported by BK21, Korea.
²Statistics Dept., Ewha Womans University, Seoul 120-750, Korea.
³IT Service Team/Information Analysis Part, LG Capital, 11F1 Nara Investment Banking Bldg., 1328-3 Seocho-Dong, Seoul 137-070, Korea.
In estimating the partworths, however, traditional estimation methods such as least squares require each subject to respond to more profiles than product attributes, resulting in lengthy questionnaires for complex and multiattributed product or service. Long questionnaires pose both practical and theoretical problems. Response rate tends to decrease with increasing question numbers, and more importantly, academic evidence indicates that long questionnaires may induce response biases.

Thus, it is desirable to develop experimental design and estimation method that estimates the partworths with shorter questionnaires. Lenk et. al. (1996) introduced a hierarchical conjoint model and showed that Bayesian analysis of the model does not require full rank individual-level design matrices and hence one may use shorter questionnaires.

The hierarchical conjoint model given by Lenk et. al. (1996) describes the variation in a subject's responses and the variation in the subject's partworths over the population as follows:

\[ y_i = X_i \beta_i + \epsilon_i, \quad i = 1, \ldots, n, \]  

(1.1)

\[ \beta_i = \Theta z_i + \delta_i, i = 1, \ldots, n. \]  

(1.2)

In (1.1), \( y_i \) is a \( m_i \times 1 \) vector of observations, \( X_i \) is a \( m_i \times p \) known design matrix, \( \beta_i \) is a \( p \times 1 \) vector of regression coefficients for the \( i \)-th experimental subject, and \( \epsilon_i \) is a \( m_i \times 1 \) vector of errors and \( \epsilon_i \sim N(0, \sigma^2_i I_{m_i}) \) independently. In (1.2), \( \Theta \) is \( p \times q \) matrix of regression coefficients, \( z_i \) is \( q \times 1 \) vector of known covariates, and \( \delta_i \) is \( p \times 1 \) vector of errors and \( \delta_i \sim N_p(0, \Lambda) \) independently. The errors \( \{\epsilon_i\} \) and \( \{\delta_i\} \) are assumed to be mutually independent. A simpler version of the above model can be given by assuming that \( z_i \)'s are all equal to 1, \( \Theta \) is the mean vector for the individual-level coefficients \( \beta_i \), and the individual variance \( \sigma^2_i \) are all equal (see Yang and Chen, 1995).

In Bayesian approach to analysis of the conjoint model, noninformative priors are often desired especially when there is no prior information on the parameters of the model or when one wants to compare Bayesian inference with classical inference. Among many possible noninformative priors, Jeffrey's prior is widely used since it is invariant under reparameterization. For the conjoint model, however, Jeffrey's prior is not easy to obtain. Moreover, since the Jeffrey's prior is improper and there are often many parameters in the model compared to observations, one needs to check if the posterior obtained from the Jeffrey's prior is